Surface integration and energy-based approaches to calculate global forces in magnetic actuators

Antônio Flavio Licarião-Nogueira¹ and Dilney Cesar B. Pereira Junior²

Universidade do Estado de Santa Catarina (UDESC), Brazil
¹antonio_nogueira@brturbo.com.br, ²dilneyfej@yahoo.com.br

Abstract — Magnetic circuit analysis and finite element solutions are used to predict the force variation of a magnetic actuator under changes in position. The limitations of the analytical approach are identified. The actuator-type device represents an application where the key to accurate force computation by the virtual work method is the body displacement selection. Further force computations show that the method of weighted Maxwell stress tensor applied to this problem reduces to a distributed Maxwell stress tensor scheme. Quick and easy to implement, the single solution energy approach gives insight to field energy storage and related force production analysis.

Keywords: — actuators, energy storage, finite element methods, magnetic forces, simulation software

I. INTRODUCTION

Design and optimization of electromechanical devices that produce force or torque are complex and highly specialized processes. The force calculation problem remains widely researched and reasonably well established, with extensive published theory and many applicational examples presented in the literature.

When the generated force is evaluated, the interest can be in determining its distribution to indicate the loading profile on a given surface or the global value that appears at the mechanical interface. Problems of industrial design like noises, deformation and vibration produced by electric equipment demand further knowledge on the theories of continuum electromechanics and magneto-elastic interactions. There has been a considerable interest in the development of force computation algorithms to evaluate the actual distribution of the forces as well as the total force between magnetized bodies in contact. Many applications of the equivalent magnetizing currents method to magnetically linear problems have been reported [1]. The equivalent magnetic charge method also carries examples of applications to linear problems [2]. Other important contributions to the art are the perturbation methods or sensitivity approaches also applied to the analysis of force density distributions and forces between bodies in contact [3]-[4].

Almost all formulations for force calculation are based on the virtual work principle or Maxwell stress tensor concept. Under the classical formulation, these methods can be applied to evaluate the total force of, practically, any device. There has been a considerable work done to assess and alleviate the numerical problems of the conventional formulations [5]. To reduce the computational effort of the virtual work method, single solution approaches have been proposed [6]. Also related to this principle, the mean and difference potentials method substantially reduces numerical errors of computed forces by subtracting energies analytically, and completely avoiding numerical differentiation [7]. With respect to the method of Maxwell stress tensor, the use of algorithms for local error estimation came as an alternative for the choice of integration contours associated to more accurate field distributions [8]. Following this, one approach based on adaptive schemes and known as the weighted Maxwell stress tensor method has been developed [9]. The method has been incorporated to general-purpose finite element programs and is used by a large community of students and designers around the world [10].

This paper addresses problems related to the calculation of global magnetic forces produced by an axisymmetric magnetic actuator using surface integration and energy-based methods. A linearized sketch of the device is presented in Fig. 1.
The actuator is cylindrical and contains a plunger that is free to slide vertically through a circular hole in the outer magnetic casing, the air gap between the two, \( g_r \), being very small. The device produces large forces through a relatively small displacement [11]. When the coil current is zero, a lower bearing supports the plunger and the air gap, \( g \), is maximum in this position. When a DC current sufficient to raise the related mass excites the coil, the plunger is lifted. An upper bearing constrains the displacement and this is the position of minimum air gap. Both bearings are made of non-magnetic material and are indicated by the letter “b” in Fig. 1. The main gap varies from 0.254 to 1.270 centimeter.

A source of magnetomotive force of 3000 Ampère-turns drives the circuit. With current flowing in the coil, a magnetic force appears to accelerate the plunger, endowing it kinetic energy. The resulting large forces combined to the small and constrained displacement permit to place this actuator in the category of limited-stroke devices. This class includes devices intended to close immediately when energy is supplied. Instead of analyzing the device’s actual motion, a comparison study is carried out considering a set of five different problems related to five different positions of the plunger. The sequence of problems numbered from one to five, starts with the position of minimum air-gap length and ends with the position of maximum gap. The actual direction of the plunger’s operation, clearly opposes this sequence. However, the observation of tabulated results and graphical representations is facilitated in this way.

II. METHODS OF ANALYSIS

The assumption of magnetic linearity is essential for the analytical approach based on magnetic circuits. The analysis is based on the equivalent magnetic circuit shown in Fig. 2 and all magnetic materials are assumed highly permeable.

\[
\begin{align*}
F &= N I \\
\phi &= \frac{N I}{R_g + R_{gr}}
\end{align*}
\]

Fig. 2. Equivalent magnetic circuit

The design parameters can be expressed in terms of the main air-gap length and the main analytical equations are presented in Table I. In these equations, \( g \), in meter, is the length of the main air gap. The length of the small side gap, \( g_r \), is constant and represents 10\% of the main gap at the end position.

<table>
<thead>
<tr>
<th>Table I</th>
<th>ANALYTICAL EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi(g) = \frac{N I}{R_g + R_{gr}} ) Weber.</td>
<td>Flux</td>
</tr>
<tr>
<td>( W(g) = \frac{1}{2} N I \phi(g) = 1500 \phi(g) ) Joule.</td>
<td>Energy</td>
</tr>
<tr>
<td>( W'(g) = \frac{1}{2} N I \phi(g) = 1500 \phi(g) ) Joule.</td>
<td>Co-Energy</td>
</tr>
<tr>
<td>( f(g) = \frac{\partial W'}{\partial g} ) Newton.</td>
<td>Force</td>
</tr>
<tr>
<td>( f(g) = -11430 \left( \frac{1000g + 0.254}{1000g + 0.254} \right)^2 ) Newton.</td>
<td></td>
</tr>
</tbody>
</table>

The numerical approach uses a simulation software based on the finite element method [12]. A unique finite element model is set and material re-identification technique is used to simulate the plunger’s movement. In the implementation, an incremental plunger movement of 25 per cent of the total displacement is used to represent the different problems. The Lua extension language adds scripting processing facilities and ensures repeatability of the results. Table II resumes the main features of the finite element model.

<table>
<thead>
<tr>
<th>Table II</th>
<th>FEATURES OF THE FINITE ELEMENT MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential solution</td>
<td>Magnetic vector potential</td>
</tr>
<tr>
<td>Mesh</td>
<td>1126 nodes , 2177 1st-order elements</td>
</tr>
<tr>
<td>Boundary conditions</td>
<td>( A=0 ), external rectangular boundary</td>
</tr>
<tr>
<td>Sources</td>
<td>3000 Ampère-turns in the coil region</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>4400</td>
</tr>
</tbody>
</table>

III. METHODS OF FORCE CALCULATIONS

The lifting force of the magnetic plunger has been computed from numerical field solutions using different methods of force prediction. The two conventional methods, namely Maxwell stress tensor and virtual work are briefly introduced in order to explain how they have been implemented to predict forces in this particular problem. This is followed by a brief description of the single solution energy approach, intended to be quick and easy to implement [6].

The analytical approach produces good estimates to a set of correlated parameters like fluxes and inductances, but is not adequate to accurately predict magnetic forces. To assess the accuracy of computed forces, another numerical method that has proved to produce accurate and consistent force predictions is used as the benchmark. The method of weighted stress tensor is a volume integral version of the Maxwell’s stress tensor that computes a weighting function by solving an additional Laplace’s equation over the air surrounding the region where the force is computed. Fig. 3 shows the contours that have been averaged to yield weighted stress tensor force result for the problem that represents the maximum gap. Depending on the weighting
function, calculated in a completely automated process, the method reduces to a distributed Maxwell stress scheme [9]-[10]. This, in fact, happens to this particular test problem, as presented later in the discussion of numerical results. Despite its complex formulation, the method has robust implementations in some finite element programs, and this greatly simplifies the computation of forces.

In the following discussion of the methods, it is assumed that discretization is appropriate to the problem. This means that finite elements are properly shaped to model field inhomogeneity. In addition, the two forms of energy will simply be referred as energy, since results are related to a magnetically linear analysis.

A. The method of Maxwell stress tensor

For the method of Maxwell stress tensor, it is possible to calculate the total force acting on a part of a device using only the field conditions on the surrounding surface or a contour of that part. For the implementation of this method, a single integration contour is used in all computations. It consists of two straight lines that partially embrace the plunger. Its horizontal portion coincides with the midgap of problem 1, which represents the plunger at the end position. The vertical portion is placed in the middle of the air layer that separates the plunger and the coil window. This portion ends close to the top of the side gap, which gives no net contribution to the force of interest.

B. The method of virtual work

The basis of the method of virtual work is the relationship between force and energy, provided by the principle of conservation of energy. The force acting on a movable part of a device may be evaluated by determining the variation of the magnetic co-energy of the entire device when a small displacement takes place. If the problem is magnetically nearly linear, co-energy can be approximated by stored energy. The main difficulty of this method is the choice of the positional displacement, which is problem-dependent and related to errors of conflicting nature. For this particular device, forces are large and subtraction of energies from small disturbances leads to the loss of one or two significant figures due to roundoff error for the operation at larger gaps. Hopefully, at this range, the variation of co-energy with position is smooth, almost linear and precisely approximated by the quotient of finite differences related to adjacent positions.

The virtual work method has been used in its classical form, evaluating the total stored energy for the plunger slightly disturbed up and down by the positional displacement to predict the force at the intermediate position. The diameter of the magnet pole was taken as guideline to choose four positional displacements, \( d_g \), representing 2.5, 5.0, 7.5 and 10.0 per cent of the total plunger displacement. The largest increment represents 2.0% of the magnet pole diameter and the force is likely to change almost linearly with distance in view of the relatively large magnet pole diameter. Following this, a set of several stored energies along the displacement vector has been computed directly from potentials, according to

\[
W = \frac{1}{2} \int A J d\Omega \text{ Joule} \quad (6)
\]

where \( \Omega \) denotes the coil region and \( J \) the conduction current densities.

C. The single solution energy approach

The single solution energy approach is physically related to the virtual work method, but relies on a reformulation of the computation sequence. Were the virtual work approach to be used, there would be at least two problems to solve, corresponding to two adjacent positions. Rather then working with two problems that would give the force at the intermediate position this method requires only one field solution. The basis of the approach is the realization that the energy change in the system with movement is broadly confined to the region where the movement takes place, and to other associated regions [6]. Its applicability is limited to some actuator-type devices and the method cannot be used, for example, to compute torque. Forces calculated by the single solution energy approach are given by a simple expression that relates the stored energy in the regions of interest to the main gap length

\[
F = \frac{W_d + W_a}{g} \text{ Newton.} \quad (7)
\]

In the above, \( W_d \) and \( W_a \) represent the energies stored in the region directly deformed by the movement and associated regions, respectively. In the present analysis, these two regions are the main air gap and the air layer between the movable plunger and the coil window.

To illustrate the above assertions, the energy changes between problems 1 and 2 are considered. The energy increase as the gap reduces is 2.023 J. About 83.5% of the energy change takes place in the identified regions of interest. In addition, 82.7% of the energy used in the calculation is stored in the gap, and this changes by 55%
with movement, as compared to the 7% change in the energy storage in the air layer surrounding the plunger.

IV. NUMERICAL RESULTS

Computed forces by the single solution energy approach are presented in Table III together with values calculated analytically and by the weighted stress tensor method, taken as the reference method. Since errors in forces computed by the method of Maxwell stress tensor and compared to the weighted stress tensor method are only a fraction of a percent, the results are taken as computationally equivalent and the data for these two methods are presented together in the last column of the table.

<table>
<thead>
<tr>
<th>Position (cm)</th>
<th>Energy approach</th>
<th>Analytical</th>
<th>Maxwell and Weighted tensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.254</td>
<td>1710.00</td>
<td>1464.18</td>
<td>1500.57</td>
</tr>
<tr>
<td>0.508</td>
<td>471.00</td>
<td>401.74</td>
<td>437.02</td>
</tr>
<tr>
<td>0.762</td>
<td>220.00</td>
<td>184.36</td>
<td>207.58</td>
</tr>
<tr>
<td>1.016</td>
<td>127.00</td>
<td>105.39</td>
<td>124.79</td>
</tr>
<tr>
<td>1.270</td>
<td>83.00</td>
<td>68.11</td>
<td>86.80</td>
</tr>
</tbody>
</table>

Table IV summarizes the errors in force predictions between the method of virtual work and the reference values, considering four different positional displacements.

<table>
<thead>
<tr>
<th>Position (cm)</th>
<th>dg=10.0%</th>
<th>dg=7.5%</th>
<th>dg=5.0%</th>
<th>dg=2.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.254</td>
<td>15.24</td>
<td>7.49</td>
<td>3.76</td>
<td>2.48</td>
</tr>
<tr>
<td>0.508</td>
<td>3.66</td>
<td>0.79</td>
<td>1.06</td>
<td>1.52</td>
</tr>
<tr>
<td>0.762</td>
<td>3.72</td>
<td>1.72</td>
<td>2.85</td>
<td>3.30</td>
</tr>
<tr>
<td>1.016</td>
<td>3.06</td>
<td>1.01</td>
<td>2.85</td>
<td>1.03</td>
</tr>
<tr>
<td>1.270</td>
<td>0.22</td>
<td>0.58</td>
<td>2.28</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Realization that energy change in the system with movement is not only confined to the region where the movement takes place, helps to explain why forces computed by the analytical approach are underestimated. The expression for the lifting force obtained by (4) only takes into account the variation of magnetic energy in the deformed region, namely the main air gap. Examination of the forces calculated by the analytical approach shows that computed forces are close to those obtained from the stress tensor method by integrating only the stresses along the horizontal portion of the contour.

Forces computed analytically are even closer to those calculated numerically by the virtual work method considering only the energy changes in the main air-gap region. Values are slightly overestimated and percentage errors between the two set of computations vary in the range of 0.34 to 0.59 per cent when the smallest positional displacement (dg=2.5%) is considered. This low level of error is expected once the analytical expression for the force has been obtained based on the virtual work principle.

Inspection of field distribution with the aid of the enlarged view presented in Fig. 4, clearly suggests that largest errors in forces computed by the method of Maxwell stress tensor are associated to the larger gaps where the effect of leakage field, especially the fringing effect is more pronounced. The line integration along the vertical portion of the selected contour gives the contribution to the total force due to the fringing field. This contribution to the total force increases from 1.98 to 11.73 per cent with the increasing gap length. For this kind of device, largest numerical errors occur at larger gaps because leakage field is associated to regions where substantial field changes take place and where accurate field distributions are more difficult to be obtained.

The problems of the energy-based methods are of different nature and the discussion starts with the presentation of the characteristic that represents the variation of the system’s energy with respect to position. The use of four different positional displacements for the five problems has resulted in a set of forty stored energies along the displacement vector. A power function given by

$$W(g) = 1.8399g^{-0.7351} \text{ J}$$

is fitted to these several values to produce the dashed nonlinear curve that appears in Fig. 5. Energy estimates related to the largest positional displacement (dg=10%) are indicated separately and connected by line segments.

Analysis of the errors associated to the energy-based methods show that the error level, in the mean, is satisfactory for the sets of computations representing problems 2 to 5. For these four problems, largest errors are 7.88 and 3.72% for forces computed by the energy approach and virtual work, respectively.

Observation of the error level associated to the computations representing problem 1 and using the virtual work method shows that forces predictions consistently improve when smaller positional displacements are used. The characteristic of Fig. 5 shows a substantial change in
energy storage at small gaps. This amounts to a reduction of 42% in stored energy in the first interval as compared to 26% and 17% in the subsequent ones. The first two points indicated in the plot quote the range where the most abrupt changes in energy take place and where the maximum force occurs. The slope of the straight line connecting these two points is visibly a poor approximation to the derivative and maximum force calculated from these points is overestimated by 15.24%. This, of course, represents the worst prediction for the maximum force. Computations based on the other sequences of stored energies produce improved estimates to the maximum force and the best match represents an error of 2.48%.

Fig. 5. Energy variation with position

For problems 2 to 4, which represent the operation at larger gaps, reduction of positional displacement from 10.0 to 7.5% results in lower numerical errors, but further reductions of this variant do not lead to any significant error reduction. For the energy estimates related to the maximum gap, the slow variation of the system’s energy with position is well represented by the approximated linear derivatives even when the largest disturbance is used. As the variant \( dg \) reduces, the ratio of energy difference to individual energies varies from 11.0 to 3.0%. This ratio should be compared to that related to minimum gap operation where reduction of the variant \( dg \) results in changes from 42.0 to 14.0%. Here, only relatively small body displacements can accurately approximate the fast variation of the system’s energy with position.

CONCLUSIONS

The work focuses on force calculations from numerical field solutions. Earlier reported recommendations on problem modeling have been considered to produce an economical, precise and flexible numerical model of the device. The study shows that, in the mean, error level of these force predictions is low and completely satisfactory. The test problem may be classified in the category of force prediction problems where the force of interest attempts to reduce the length of the air gap and where the relative contribution of leakage field energy storage to the force is readily identified. Problems of this type are easier to solve than, for example, those involving overlapping poles and problems where the force of interest has to be calculated in the presence of a force of much greater magnitude in the orthogonal direction.

REFERENCES