Abstract — The paper describes the authors’ approach to teaching electric power equipment CAD techniques to undergraduate students pursuing a degree course in Electrical Engineering. Details of one experiment undertaken by the students are given. The experiment focuses on the calculation of terminal inducances from numerical field solutions. The simulation tasks are easily adapted to the analysis of a greater varied of electrical equipment.

Keywords: inductance, energy storage, engineering education, finite element methods, simulation software

1. Introduction

Design and manufacture of electric power equipment is a challenging task and the integration of electrical, mechanical and thermal designs turns out to be a highly specialized process subject to many requirements other than electromagnetic. In the past three decades, an increasing number of manufacturers rely, to a varied extent, on electromagnetic computer-aided design (CAD) systems in their electrical design processes. Much depends on existing facilities and the nature of the device. A designer of direct current machines, for example, is more likely to use finite element-based CAD systems than the designer of induction motors. This kind of machine is more difficult to analyze using finite elements and requires a judicious balanced blend of classical and numerical methods. All these factors have to be considered when designing CAD experiments, as its relevance to the students’ future work depends largely on the facilities which will become available to them in employment.

Today’s increasing access to field simulation software, allied to the ease-of-use of the new generation of field simulators has motivated the creation of a laboratory of simulations. The philosophy underlying the simulated laboratory component of subjects based around electromagnetics is described in [1]. The experiments place emphasis on demonstrating how to set up field problems for solution and how to obtain critical design parameters from field solutions. In other words, students are expected to become familiar enough with the sequence of postprocessing operations to be able to use field simulators with minimum supervision. At the State University of Santa Catarina (UDESC), the energy conversion course consists of 60 hours of lectures and 30 hours of supporting laboratory work. The five experiments of the conventional laboratory are devoted to the determination of low-power transformers’ performance. In view of the new laboratory of simulation, a 6-hour introductory short course has been developed to teach the basic concepts of the finite element technique. The object of this short course is to demonstrate how differential equations describing some physical systems can be solved by potential energy minimization, and how the accuracy of the solution depends on the order of the trial function. This is followed by a series of 6 laboratory experiments where analytical calculations and field computations are made on simple physical devices.

As a result of the work on the laboratory of simulation, the students get acquainted with the benefits of simulations such as visualization of abstract concepts and inspection of effects such as magnetic saturation, force distributions and parasitic effects. Another important result is to show the students the limitations of electromagnetic field simulators, as they are only part of the real design projects.

2. Experiment: inductance and magnetic linearity

In electronic power equipment, inductors are used to smooth out ripple voltage in dc supplies, an application where they carry direct current in the coils. Most practical designs include an air gap in the magnetic core to prevent magnetic saturation and therefore control the inductance value under a wide range of operating conditions. The definition of the number of turns, geometry of the magnetic core and length of the air gap has to take into account the following interrelated factors: desired inductance value, direct current in the winding and ac voltage across the winding.

When measurements or numerical simulations using increasing values of excitation show that the value of inductance remains nearly constant up to high excitation levels, the device is called a linear inductor. If, on the other hand, the inductance value drops with the increasing of excitation, the device is a nonlinear inductor. This nonlinear effect is reflected not only on the low inductance value but also as a source of noise and mechanical vibration. Following the different definitions for the phenomenon of inductance, this laboratory guide
explains how the air gaps included in the magnetic core control the inductance value of a simple magnetic-core inductor. The main features of the analysis are the effects on inductance values resulting from

- different techniques for calculating the inductance numerically;
- inclusion of short and large air gaps in the magnetic core;
- proportioning of the energy stored in magnetic and non-magnetic regions.

3. Techniques of inductance computation

The inductance of a given device can be defined, either in terms of its magnetic stored energy or in terms of the number of flux linkages of the winding [2]. If \( L \) denotes the total value of the inductance, and \( L_r \) the inductance value rated with respect to the number of turns squared \( n^2 \),

\[
L_r = \frac{L}{n^2}.
\]  

In the following development, values for the total inductance are expressed in terms of the device’s terminal current \( i \), while those for the rated inductance are expressed in terms of the total excitation in ampere-turns \( I = ni \).

The first technique for inductance calculation is based on the concept of flux linkages and uses the following expressions

\[
L = \frac{n\phi}{i} = \frac{n\phi}{I/n}.
\]  

where \( n \) is the number of times the winding links the flux \( \phi \). Calculation of the rated inductance is then

\[
L_r = \frac{L}{n^2} = \frac{\phi}{I}.
\]  

The other technique for the calculation of inductance based on the concept of flux linkages uses the following expression:

\[
L = \frac{\int A J \, d\Omega}{I^2}.
\]  

where \( \Omega \) represents the area occupied by the current-carrying regions; \( A \) and \( J \) represent the distributions of magnetic vector potential and current density, respectively. The rated inductance turns out to be

\[
L_r = \frac{\int A J \, d\Omega}{I^2}.
\]  

If the inductance is defined in terms of the magnetically stored energy \( W \),

\[
L = \frac{2W}{I^2}.
\]  

The rated inductance, in this case, is given by

\[
L_r = \frac{2W}{I^2},
\]  

and the stored energy \( W \) is evaluated by

\[
W = \int_{\Omega} \left( \int H(b) \, db \right) \, d\Omega.
\]  

where \( \Omega \) represents the entire problem domain.

When detailed specifications of the winding are not available, calculations based on expressions for the rated inductance turn out to be a convenient way of deriving information related to inductance. In this case, the coil region may be represented as a single-turn massive conductor. The field solution for this single conductor carrying a total current of \( I \) amperes can be used later to calculate the inductance of the equivalent \( n \)-turn winding carrying a terminal current of \( I/n \) amperes. If \( \phi \) is the flux obtained from the potential solution with a single-turn conductor, the quotient \( \phi/I \) should be multiplied by \( n^2 \) to give the correct value of inductance.

4. The magnetic-core inductor

The basic configuration of the inductor used in the study consists of a single coil or busbar wound around the central limb of a rectangular magnetic core. The main geometrical dimensions are shown in Fig. 1. The size of each window is 20.0 cm by 20.0 cm and the busbar is placed in a rectangular region of 12.0 cm height and 6.0 cm width. A non-magnetic insulating layer of 4.0 cm separates the conductor from the magnetic core. The depth of the device is 7.5 cm.

\[\text{Figure 1- Sketch of the inductor, dimensions in centimeter.}\]
The symmetry of the core and winding allows analyzing only one-half of the device, as shown in Fig. 2. The finite element model includes several regions that allow extra flexibility for altering material properties, changing the placement of the winding and facilitate the analysis of the results. The light gray area that appears in the right side limb of the core, for instance, is a union of artificial material boundaries that enables the inclusion of air gaps with different lengths. Several air layers in the core window and around the right side limb permit to quantify the energy storage of the leakage fields.

5. Numerical results

The study includes three sets of field solutions corresponding to three configurations of the inductor. The first configuration consists of a highly permeable core without air gap, while the other two configurations include air gaps in the outer limbs. The idea is to show that air gaps of different lengths may be included in the magnetic circuit to control the inductance variation when the device operates over a wide range of excitations. The lengths of the air-gap layers of the second and third configurations are 0.25 and 2.50 cm, respectively. For each configuration, potential solutions are obtained along a range of excitations that vary from 0.15 to 15.0 kA-turn.

Inspection of saturation is made by observing how the device’s input energy supplied by the electric source is partitioned into proportions of magnetically stored energy and co-energy. Under magnetically linear conditions, the input energy is divided into equal amounts of stored energy and co-energy. The average of the energy and co-energy values is commonly referred as the linear energy $W_L$ [6] It is numerically equal to half of the input energy and may be calculated directly from potentials as

$$W_L = \frac{1}{2} \int_{\Omega} A J d\Omega,$$

where $\Omega$ denotes the coil region. Under saturated conditions, the input energy is no more divided into equal amounts of energy and co-energy. As a result, the characteristics that represent the variation of stored energy and co-energy with changes in position and/or excitation become nonlinear. The characteristic that represents the mean of energy and co-energy values, however, exhibits incremental linearity [7]. Magnetic saturation is reflected in the difference between inductance values calculated from different techniques. Inductance calculations using the linear energy produce the same values as the inductance calculations from flux linkages, as shown later in this guide.

5.1. The gapless core configuration

The field solution obtained for the gapless-core configuration using 0.15 kA-turn, the lowest excitation, is firstly examined. The illustration of Fig. 3 shows the averaged flux densities that cross the core limbs, coil region and its adjacent air space. The illustration shows that flux densities are everywhere very low and most of the flux flows through the magnetic trajectory. Clearly, for this excitation value, the device operates on very low level of magnetic saturation.

The solution computed for the excitation of 15.0 kA-turn shows that flux densities in the core rise by a
factor of about 3.0 when compared to the other solution. Flux densities in the coil region and in the air space of the window are, in the mean, 10.0 times their values at low saturation level. However, most of the flux continues to circulate through the magnetic core and leakage flux still represents a fraction of a percent of the total flux.

The distribution of different forms of energy for an increasing level of excitation is presented in Fig. 4. To ease the visualization of the excitation level where the threshold of saturation occurs, the characteristics are displayed along a narrower range of excitation, from 0.20 to 1.0 kA-turn. Analysis of the characteristics presented in Fig. 4 shows that, the inductor can be considered a magnetically linear device for operation at low excitations, up to about 0.30 kA-turn.

As the excitation increases beyond this value, the difference in values of the two forms of energy becomes more pronounced, indicating an increasing level of saturation in the magnetic core. As a result of the operation under increasing levels of magnetic saturation, inductance values decrease by about 28 times, as shown in Fig. 5.

Inductance values for the gapless core inductor calculated for the two extreme levels of excitation, using the three different techniques just described are resumed in Table 2. Presented results are the rated inductances $L/n^2$ for a single-turn winding. If the number of turns is known, these values should be multiplied by $n^2$ to give the correct inductance value.

5.2. The gapped-core inductors

In order to minimize the changes in inductance values under varying operation conditions, many inductor configurations include one or more air gaps in its magnetic core. The original, gapless configuration of the inductor has been modified to include air gaps in the two outer limbs. The lengths of the included gaps for the second and third configurations are 0.25 and 2.50 cm, and they will be referred as short and large gaps in the following discussion. These gaps represent 0.25 and 2.5 percent of the mean flux path length, which is 1.0 meter.

As a result of the included gaps, the device’s magnetic linearity is extended over a wider range of excitations, at the expense of a reduction in inductance values at low excitation levels. To drive the gapped inductors into the saturated region, the required excitations are now 2.0 and 11.0 kA-turns for the second and third configurations, respectively. These excitations are about 7 and 37 times larger than that required to saturate the gapless-core inductor. Tables 3 and 4 summarize the
inductance values of the two gapped-core inductors for the two extreme levels of excitation.

**Table 3- computed inductance: core with short gap.**

<table>
<thead>
<tr>
<th>Excitation (kA-turn)</th>
<th>L via flux linkages (μH)</th>
<th>L via stored energy (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eq. 3</td>
<td>Eq. 5</td>
</tr>
<tr>
<td>0.15</td>
<td>3.465</td>
<td>3.461</td>
</tr>
<tr>
<td>15.0</td>
<td>0.800</td>
<td>0.814</td>
</tr>
</tbody>
</table>

**Table 4- computed inductance: core with large gap.**

<table>
<thead>
<tr>
<th>Excitation (kA-turn)</th>
<th>L via flux linkages (μH)</th>
<th>L via stored energy (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eq. 3</td>
<td>Eq. 5</td>
</tr>
<tr>
<td>0.15</td>
<td>0.753</td>
<td>0.744</td>
</tr>
<tr>
<td>15.0</td>
<td>0.672</td>
<td>0.667</td>
</tr>
</tbody>
</table>

For the configuration with short air gap, the figures of Table III show that the variation in inductance with excitation is smaller than that for the configuration without air gap. In other words, the inclusion of a short gap represents a first step in reducing the changes in inductance values. A definitive improvement however has only been achieved with the third configuration. For this inductor, with a relatively large gap, all inductance calculations are in excellent agreement for the two operating conditions summarized in Table 4. The computed inductance values for the unsaturated condition are, in the mean, only 1.15 times larger than that for the saturated case.

The graph of Fig. 6 shows the percentage of magnetically stored energy in different regions of the inductor with short gap operating over an excursion of excitations up to 50.0 kA-turns.

Observation of the characteristics of Fig. 6 shows that, for low excitations energy storage is dominated by the magnetic field in non-magnetic regions associated to the coil, air gap, and fringing flux around the gap. However, as the excitation increases beyond 2.0 kA-turns, the rise in the energy stored in the magnetic core becomes more pronounced than the correspondent decrease in the energy stored in the main gap and coil region. As a result, the energy storage in the saturated core starts to represent an increasing proportion of the device’s total energy. For an excitation of 18 kA-turns, the energy of the saturated core represents half of the total stored energy. This helps to explain the inductance variations for the inductor with short gap.

**Figure 6- Proportion of stored energy, core with short gap.**

Fig. 7 shows the flux density plot for the inductor with large gap operating at 50.0 kA-turns.

**Figure 7- Plot of flux density, excitation of 50.0 kA-turns.**

With the aid of this plot, it is possible to observe that a considerable amount of the device’s total energy is due to the energy storage in the leakage fields and this is reflected in the device’s magnetic linearity. A careful observation of the graph presented in Fig. 8 corroborates that observation and helps to quantify the proportions of energy storage in magnetic and non-magnetic regions. Fringing flux around the gap, for example, represents more than 30% of the total energy along first half of the range of excitations. The increase in the energy stored in the magnetic core is now relatively slow and this energy storage reaches only 21% of the total device’s energy when the excitation is maximum. This helps to explain why changes in the relative permeabilities of the saturated core do not affect much the inductance of inductors containing relatively large air gaps.
6. Conclusions

The paper describes the authors’ experience in creating a laboratory of simulations to support the energy conversion course. Gives details on one experiment undertaken by the students. The pedagogical investment forms part of a continuously assessed undergraduate coursework.

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